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AUTHOR Frid, Sandra; Malone, John

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ABSTRACT

Specific mechanisms of classroom interactions by which teachers and students conjecture, criticize, explain, test, and refine ideas and procedures is what is referred to as "negotiation of meaning in the mathematics classroom." Elaboration of the form these mechanisms take in practice, and hence investigation of classroom constructions of meaning, is required if the relationship between student classroom experience and student constructed meanings is to be understood in a way that will inform instruction. The purpose of this study was to investigate the relationship between students' classroom experiences and the manner in which they construct mathematical meanings. Two intertwined research foci guided this investigation: (1) how students make sense of and utilize mathematics concepts and operations, and (2) the social context within which teachers' and students' individual contributions play a role in the sense making and utilization of mathematics concepts and skills. Data were collected from classroom observations and videotaping sessions and from subsequent video-stimulated interviews with six students in each of two year 5 classes in two different government schools in the metropolitan area of a large city in Western Australia. Analysis revealed the existence of four primary sources by which students determined the meaning or correctness of mathematical activity: the teacher, intuition, familiarity, and procedural knowledge. Second, in relation to the social level, the teacher emerged as playing the most valued role in the sense making and ratifying of procedures or answers. Contains 13 references. (MKR)

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Sandra Frid and John Malone

Science and Mathematics Education Centre
Curtin University of Technology
PO Box U1987
Perth, Western Australia
Australia 6001

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Negotiation of Meaning in Mathematics Classrooms: A Study of Two Year 5 Classes

Sandra Frid and John Malone
Curtin University of Technology

The purpose of this study was to investigate the relationship between students classroom experiences and the manner in which they construct mathematical meanings. Two intertwined research foci guided this investigation: (i) how students make sense of and utilise mathematics concepts and operations, and (ii) the social context within which teachers' and students' individual contributions play a role in the sense-making and utilisation of mathematics concepts and skills. Data was collected from classroom observations and videotaping sessions and from subsequent video-stimulated interviews with 6 students in each of two year 5 classes. Results discussed here are those from analysis of the interviews. This analysis revealed the existence of four primary sources by which students determined the meaning or correctness of mathematical activity: the teacher, intuition, familiarity and procedural knowledge. Second, in relation to the social level, the teacher emerged as playing the most valued role in the sense making and ratifying of procedures or answers.

INTRODUCTION

Assumptions underlying the ways mathematics education researchers view knowledge, cognition, learning and teaching have shifted dramatically in recent years. This can be seen most clearly in the lack of specific reference to constructivist theories of learning in papers offered to the 1984 conference of the International Congress of Mathematics Education. By 1987, "Constructivism" was mentioned in the titles of three of the four keynote addresses to the annual conference of the International Group for the Psychology of Mathematics Education. Critical re-examination and re-conceptualisation of learning situations in mathematics education (and in science education) have led to widespread acceptance of constructivist theories of learning (Ernest, 1989). A substantial body of classroom data is emerging internationally to substantiate a constructivist picture of the learning process (for example, see Perkins & Simmons, 1987; Confrey, 1990). The study reported here was designed to investigate classroom learning from a perspective coherent with constructivist theories.

Apart from providing an interpretative framework for the analysis of classroom learning, constructivist theories have had little actual influence on mathematics instruction. Specific mechanisms of classroom interactions by which teachers and students conjecture, criticise, explain, test and refine ideas and procedures is what is referred to by the concept of "negotiation of meaning in mathematics classrooms". Elaboration of the form these mechanisms take in practice, and hence investigation of classroom construction of meaning, is required if the relationship between student

classroom experience and student constructed meanings is to be understood in a way that will inform instruction. Hence, the goal of this project was:

> To investigate the relationship between students' classroom experience and the manner in which they construct mathematical meaning.

Within this overall goal were two intertwined research foci:

(i) the individual level at which students make sense of and utilise mathematics concepts and operations, and

(ii) the social level within which teachers' and students' individual contributions play a key role in a social making sense of and utilising mathematics concepts and skills.

The two research foci (the individual and social levels) arise naturally as a consequence of the fact that learning theories which fall within the constructivist school invoke the negotiation of both academic content and social context meanings as inevitable characteristics of classrooms, rather than as pedagogical options. Thus, the dual perspective of sources of conviction (Frid, 1992) and classroom consensus processes (Clarke, 1986) was selected as a guiding interpretative framework for the study. Sources of conviction refer to how one determines facts, legitimacy, logicality, consistency and accordance with accepted mathematical principles and standards (i.e. academic content meanings). Consensus processes are typified by group compromise, refinement and accommodation. They are taken to be those interactions whereby conjectures and arguments arising in classroom discourse are compared and assessed (i.e. development of social context meanings). Since the dual analytic perspective was located firmly in the classroom context, it facilitated examination of the symbiotic relationship between social and individual construction of meaning: symbiotic in the sense of being mutually dependent and mutually supportive.

METHOD AND DATA SOURCES

Since this study was concerned with the context of learning and related descriptions of teachers' and students' mathematical interpretations, qualitative research methods predominantly were employed. This approach is in line with current educational research practices as they shift away from quantitative, quasi-scientific experiments so that researchers can more explicitly document and analyse the experiences of teachers and learners in the broad encompassing social and academic complexities of classrooms (Clarke, 1985, 1992). In particular, this project adopted an inductive reasoning approach (Glaser and Strauss, 1967; Powney and Watts, 1987) for analysis of data collected by videotapes of classroom lessons and video-stimulated recall interview techniques (Keith, 1988; Meade & McMeniman, 1992). There was therefore ongoing interaction between theory and research as the

analytic framework (sources of conviction and consensus) and emergent themes were refined. The emergent key patterns were therefore grounded in primary data.

The two year five classes chosen for the study were in two different government primary schools in the metropolitan area of a large city in Western Australia. The classes included students of low and middle socio-economic levels, as well as some students of minority backgrounds (aboriginal and Asian). Nine visits were made to each class with six lessons videotaped for each class between the months of March to August 1993. Following each lesson, two students were interviewed using a video-stimulated recall technique. These students were selected by the teacher so that the entire sample included a balance of males and females as well as students representative of a range of achievement levels in mathematics. Other student characteristics guiding selection included high versus low verbalisation and participation in class activities. The overriding principle guiding sample selection was that a rich diversity of cases be provided. A process of matrix sampling of students was used to ensure maximum diversity of sample cases, with a guarantee that each student was interviewed twice. Thus, a total of 24 interviews were conducted with students, each interview of a length of 30 to 45 minutes.

The protocol for the video-stimulated interviews asked students to respond to episodes in a 15 to 20 minute segment of the video. The episodes included teacher actions and utterances, other students' actions and utterances and the interviewee's actions and utterances. Choice of the 15 to 20 minute classroom segment was determined by the researchers so as to maximise the number of episodes which could be discussed in the interview. Students were asked to respond to a selection of the following questions in relation to the episodes:

Interviewee actions and utterances:
What were you thinking at that moment?
Why did you do that?
Why did you say that?

Teacher actions and utterances:
What do you think the teacher meant by that?
Do you feel that you understood what the teacher was saying?
Did you agree with what the teacher was saying? (Why? or Why not?)
How do you know when you understand something? (Can you give me an example?) How did you know in this instance?

Other students' actions and utterances:
What do you think that student (on the videotape) meant by that?
Do you feel that you understood what the student was saying?
Did you agree with the student at that point? Why? (or why not?)

After the videotape had been viewed students were also asked for general reflective comments on their mathematics class. They were asked to comment on what they felt

was the most important thing that happened in the lesson, what they felt they learned in the lesson, what convinces them that something in mathematics is right, when and why they change their minds during lessons, and what they felt the other students learned in the lesson.

The videotaped classroom lessons and video-stimulated recall techniques were analysed to obtain the following information:

(1) student perceptions of their own constructed meanings in the course of a lesson,

(2) students' sources of conviction for construction of their mathematical meanings,

(3) the extent of class consensus as to socially constructed meanings and the process whereby consensus was (or was not) achieved, and

(4) the individuals, experiences, arguments or actions in which students felt mathematical (academic content) authority resided.

Interviews were recorded on audio tape and later transcribed to form part of the data set. Analysis proceeded cyclically through identification and verification of relevant and prevalent episodes present in the data. To add to the reliability of the analysis of the interview data an independent researcher was asked to read through the transcripts to categorise student statements that she identified as related to the following questions:

How do students make sense of mathematical concepts and operations?

What is the teacher's role in this process?

What are the sources of student understanding of mathematics?

Do teachers and students negotiate meaning? If so, how?

Do students have a conception of consensus? If so, how is it manifested?

RESULTS AND RELATED DISCUSSION

As mentioned earlier, the results and discussion reported here are those from analysis of the interview transcripts. First, in relation to the individual level of making sense of and utilising mathematics concepts and operations (research focus (i), sources of conviction), interview data revealed the existence of four primary sources by which students determined the meaning or correctness of mathematical activity: the teacher, intuition, familiarity and procedural knowledge. Second, in relation to the social level within which teachers' and students' contributions play a role in making sense of and utilising mathematics concepts and operations (research focus (ii), consensus processes), one primary feature emerged from the data: the teacher. Each of these areas will now be more clearly defined and each will be elaborated upon and supported by primary data from the interview transcripts.

Students' sources of conviction

To determine students' sources of conviction, categories in the data were determined by identifying the sources of student understanding of mathematics, how students make sense of mathematical concepts and operations and the role of the individual, the teacher and other students in this process.

Related to these three issues students perceived authority in knowledge and procedures to reside primarily in the teacher, with the individual's sense of familiarity with, intuition about or procedural knowledge of concepts and operations serving generally as secondary sources of conviction.

Teacher as a source of authority

Students saw the teacher as the primary authority on the meanings of mathematical concepts and the correctness of operations or answers. Within this context they saw the teacher as the individual with access to the "truth" or "correctness" of particular pieces of mathematics. This point is particularly clear in the following interview extracts:

(Anna)
Interviewer: Would you agree with what the teacher was saying?

A: Yes.

I: Why?

A: Because teachers are normally right.

I: Oh, I see. What makes you think they're normally right?

A: Um. Because they had to go through a school, teaching school, and they learn all about decimal points.

(Yvonne)
I: Let's start with these ones. What would convince you if you were doing those that you were doing them right?

Y: Well first of all Mr Y tells you how to do it, so like it's pretty easy cause like I'm learning my ten times tables, so you just do your times table. And Mr Y tells you what to do next.

(Walter)
I: Okay. So do you agree with what Neil had put there?

W: Yes.

I: Okay and why do you agree?

W: Because it was exactly the same as what Mr Y had on the board. And the right numbers and everything.

Both Anna and Yvonne take what the teacher says as "right" or "how to do it" or "what to do next" as a reason to be convinced of the correctness of their answers or procedures. Similarly, Walter takes agreement with what the teacher says as a basis for knowing of correctness. These aspects of the students' interview responses do not in themselves imply they do not make any use of their own sense of what seems correct. However, it appeared when students were probed about how they knew they understood something, that even when they referred to their own sense of something making sense they relied on the teacher as an initial or an ultimate source of the coherency. For example:

(Anna)
I: How did you know you understood?

A: Because the teacher explained it um she explained it really clearly. . . . And when the teacher explains it clearly I um then I think I'm going to get it right.

(Rachel)
I: Okay, how do you know that's all right?

R: Oh, because I have faith in my teacher and I believe him.

I: . . . How do you know when you understand something?

R: Because it just looks right to me and it feels right to me. It feels right. And I can always check it with a teacher if I'm not that sure.

I: . . . When do things look right?

R: Things look right when I just look at them and they, and also another thing that told me it was right was that Mr Y walked around, looked at my answer and said yeah good answer. So when the teacher does that then I know for sure that it's almost right or pretty close to right. Or maybe even is right.

Anna relied on the teacher to explain the mathematics to her, and if she considered the explanation to be "clear" then she felt she understood. In comparison, Rachel refers to a feeling or intuitive sense of knowing when she understands. However, for her it is evident that, as with the other students, the teacher is an ultimate source for determining the validity or correctness of procedures or answers. This same view that the teacher's explanation is the primary guide by which to proceed to get right answers is evident in Lisa's reasons for knowing when she understands something:

(Lisa)
I: Well when you're working through with your maths how do you know when you understand it?

L: Well Mrs. K does go through it and when everybody um you know yells it out and she says it's right then I think of that number and if she says it's right then I know I understand it. And if it's wrong, when she points it out, what's right, I do get, don't really understand it, but I go over it a few times and then I do understand it.

I: . . . Are there times before you're told answers that you already feel that you don't understand it?

L: Ah yeah, I will listen before I try it out myself and find out if I understand it or not. But if I don't she, she will come and if she sees that I've got um it wrong then she'll come and tell us if we, explain it to us and we'll find out if we understand it or not.

Lisa's last comment about the teacher explaining "it to us and we'll find out if we understand it or not" is particularly revealing of her sources of conviction. It seems to indicate that Lisa does not see herself as a source of the determination of her understanding. Rather, she finds out if she understands or not by having the teacher tell or explain "what's right". Unfortunately, as can be seen in the following extract from the interview with Alison, students do not always know for themselves what the teacher's reasons are for particular explanations or performance of procedures:

(Alison)
I: So why do you think the teacher said that? Any idea why?

A: Because um he wanted us to get back to the number we started with without having a decimal point in it.

I: And why do you think he wanted that?

A: I don't know.

I: You don't know? Do you think he had a reason?

A: (long pause) Yes.

I: Do you have any idea what that reason is?

A: Not really.

In this episode Alison is referring to a task the teacher had the students doing with a calculator that involved multiplying a number by ten repeatedly to move the decimal point. For Alison it appears that, although she knew the final goal of the exercise, she did not know what the teacher's reasons were for performing the given operation of multiplying by ten. However, as revealed in the following extract,

students also at times did feel they knew why the teacher proceeded in a particular way:

(Walter)
I: Okay. So why do you think he did it in that way?

W: Cause it was, it was an easy way for Elaine to understand, to try and understand that it was smaller.

To summarise, it can be said that students saw the teacher as the primary authority on mathematical meaning. However, it must be noted that in spite of the apparent dominance of or reliance on the teacher's knowledge base, students did express related personalization or internalization of knowledge, as discussed in the upcoming two sections.

Intuition

Students included their own personal feelings that something "just makes sense" as a means by which to determine appropriateness of a procedure or answer. For example, Nathan speaks in the following interview excerpt of having a "feeling" and being able to "sort of know" when he understands or has something right:

(Nathan)
N: Um. Well I have a feeling and when the teacher puts it up on the board then I sort of like, she has half the question and I sort of know it's right already. (pause) Before she starts. But when she's writing it on the blackboard and I've already got my answer down and she's just gone through it half way already, ah, put the numbers down already, I sort of like know it's right.

I: Do you have any idea how you know?

N: Um. I can sense it really.

It was not only an intuitive "sense" which some students brought into play in explaining how they knew when they did or did not understand things, but also how something looked or sounded. For example:

(Alison)
I: Did you agree with him?

A: Kind of.

I: Can you say more?

A: He said that because um it just didn't sound right. It just didn't sound right.

(Walter)
I: How do you know when they're wrong?

W: (pause) I don't really know. You can sort of just see that they're wrong.

I: In what way can you see?

W: Like you might have a thing up on the board and say they were doing the racing thing again, you could see that they're wrong cause they've got the wrong blocks. And maybe they might not have the right amount in numbers. And you could see that way.

Alison expresses a sense that she felt strongly that something didn't "sound" right, even though she did not have a reason for why it did not sound right. In comparison, although referring to another physical sense, that of seeing rather than hearing, Walter is able to give a reason for why he could "see" something was wrong. However, it was generally the case that students were seldom able to articulate reasons why something intuitively made sense. This finding is typified in the following extract from the interview with Walter:

(Walter)
I: Okay. How do you know that you understood it?

W: Mm. I don't really know. I just do.

Although it is not evident in this short interview extract, Walter responded to the request to explain how he knew in a way that indicated he had never been asked such a question before. That is, he seemed quite surprised by the question. Most of the students reacted and responded similarly to Walter to metacognitively oriented questions. Their lack of metacognitive awareness cannot however be automatically attributed to an inability to think metacognitively about their work. It might be that such aspects of working with mathematics have never been communicated to or stressed or practised with them, in which case it would be unreasonable to expect then to somehow have naturally developed such skills and awareness.

However, in spite of students' general lack of facility to articulate and explicate reasons for their knowledge, they were able to associate "familiarity" with a personal sense of knowing. That is, one aspect of their learning that consistently arose in relation to metacognition was their awareness of when things were familiar, or had been "learned before". This point will be outlined next.

Familiarity

Students' recognition of mathematics concepts or operations as things they had seen or done earlier in the year or in a previous school year gave them a sense of knowledge of and competence with related ideas and skills. For example, in the

following interview extracts students refer to previous experience with particular ideas or procedures:

(Lisa)
I: How do you know when it looks easy?

L: Well when it looks easy it means I've seen it before and I know what to do.

(Rachel)
I: And why do you agree?

R: I agree because it makes a lot of sense to me. Because I've heard it before. And if, I find it really easy to understand things which I've heard before.

(Yvonne)
I: Now when you feel you understand it, what tells you that?

Y: Cause like you can remember and it's clear, like um (pause) um, like it's in the back of your head all the time and you can remember it. And it's quite clear to you. So it comes back.

(Alison)
I: When you were working through them, did you know you were getting them right?

A: Kind of.

I: Kind of. How did you know?

A: Um. Cause I've done them heaps of times.

I: You've done them heaps of times.

A: And I learned them.

(Anna)
I: Okay. Do you feel you understood what was going on there?

A: Yes.

I: Why?

A: Because um we did it in a test once and I knew all about it.

I: Did you understand today?

A: Yup.

I: And um how is it you knew you understood it?

A: Because I've learnt it before in my other school. . . . Because I learned it in my other school.

In these extracts the notions of remembering and having learned, seen, or done something before are predominant. The resultant sense for the students of "knowing what to do" gives the students a sense of understanding. However, it was not always clear to what extent an external observer also might describe some of the students' actions as indicative of "understanding". In particular, there were numerous instances in which it appeared that students equated the word "understanding" with instrumental knowledge as described by Skemp (1987). That is, they had "rules without reasons" (p. 153), or more specifically, they were able to perform procedures to obtain answers.

Procedural knowledge

When students believed they knew the correct procedures to yield correct answers they felt they had both understanding of and skill with the related mathematics. This aspect of their mathematics learning can be seen in the following excerpts from the interviews with Amanda, Yvonne and Alison:

(Amanda)
I: Now when you working through these first ones, how did you know when you were right or not?

A: Um. Well, I colour, it says to colour or shade in five pieces out of six. So I colour in one sixth, another sixth, another sixth and another sixth and another. So I've got five sixths that I coloured in. One left over equal, equal the six pieces. I have five coloured in. . . . I know something's right, like I know my times tables. And then like for a times table thing, if I know my times I know I have got it right. And if I practice them at home and I learn them, I think I've got them right so I just go through my times tables and go one times four is four, two four's is eight, and so on until I make sure I've got it right.

(Yvonne)
I: Well if it didn't make sense, how would you feel then? How do you know when it doesn't make sense?

Y: I'd be confused and I wouldn't know like how to do it, and like I'd have to ask again.

I: You'd be confused and you wouldn't know how to do it and you'd have to ask again. Whereas here you know how to do it and you went right through it.

Y: Yeah.

I: When you say I'd be confused, can you say more about that? How do you know when you are confused?

Y: Um.

I: Tough question?

Y: You just like don't know how to do it. And like you should know how to do it.

(Alison)
I: So how did you know that you didn't understand?

A: I didn't know what to do.

I: You just didn't know what to do? Did it feel different than when you do understand?

A: Yes.

I: How did it feel different?

A: Well then I know the answer straight away and I don't sit there for a long time.

The sense of "knowing what to do", that is, knowing a procedure is prominent in these extracts. In fact, it was often difficult to probe further to determine if the students had conceptual understandings underlying their procedural knowledge. Not only did explanations tend to rely on detailed descriptions of procedures, as with Amanda in the above extract, but they tended to reveal that students saw procedural knowledge as a primary goal of their mathematics learning. They sometimes referred to knowing how things "fit together", but were often not able to articulate this "fitting together" in ways an observer might say are indicative of conceptual understanding. For example, in the following interview excerpt, Amanda specifies getting "right answers" as reflective of understanding. Although she also mentions it as being more than copying in that it involves understanding "how everything works", she does not elaborate upon what she means by this. Even after further probing by the interviewer to attempt to explicate Amanda's conceptions of understanding, little more was revealed:

(Amanda)
I: Do you have any sense of how you know, or what does it feel like to have the right idea? How do you know when you have the right idea?

A: Well, (pause) um I get the answers right. Um not like just like copying or anything. I do them and I get them right. And then I understand how everything works.

In summary, students associated understanding with being able to complete procedures and obtain correct answers. To what extent they also had achieved knowledge of the underlying structure of or relationships between concepts was difficult to determine. In fact, what must be noted in relation to this point is that it also was not clear to what extent students aimed to achieve procedural versus more conceptually based knowledge. These points will be discussed further in the concluding section of this report.

Classroom Consensus Processes

Examination of the transcripts for whole class context examples of how students determined mathematical meaning or correctness revealed that, as with the individual level, students accepted without question that teachers' comments, guidelines, evaluations and decisions were legitimate and correct. Within this social realm, peers were not heavily utilised, except when class majority was cited as a component of particular decisions, or when the teacher was not available and a peer had to be approached for assistance to proceed through a particular set of exercises.

These two primary features of consensus, the dominant role of the teacher and the occasional intervening role of peers are outlined next.

Teacher and student roles in consensus processes

The roles of teachers and students within the class, as perceived by the students, will be discussed by focussing on a small sample of interview excerpts and highlighting factors that these excerpts suggest. First, the following extract from the interview with Lisa:

(Lisa)
I: Do you ever ask any of your friends to help you with it?

L: Um. No.

I: Do they ever ask you for help?

L: Not really. They usually just wait for the teacher to tell them.

Lisa clearly sees the teacher's role as one of "telling" students and the roles of herself and her peers do not include helping one another. Though not as strong in her views that peers have no role in mathematics learning, Rachel also expressed that she felt peers were not important to her mathematics learning:

(Rachel)
R: When another student says it I'm a bit dubious because they might just be saying that. Just to make me change my answer say. Cause some students can trick you and I've been tricked quite a few times. Cause my neighbour said that this was wrong and so ah and it was right. And I changed it.

I: Who do you feel you rely on?

R: Now I have found it really better to rely on teachers and calculators. Cause they have a better and more accurate answer than my peers. . . . Because like, say Mr Y went around and he said, and he's looking at our answers to help me, make me change my mind. Sometimes he can say like say I had the wrong answer he can say have a look at it. If it doesn't look right to you then it probably isn't right. And then I look at it and I realise it doesn't look right so I try again.

I: So am I right to say Mr Y points at something for you, or points it out?

R: Yup. He points it out so that we know what we've done wrong so that we don't make the same mistake twice.

Rachel's words reveal that she places little value on the role of her peers in classroom processes. Instead, she relies on the teacher or a calculator to validate answers because "they have a better and more accurate answer" than peers. Thus, from Rachel's perspective classroom "negotiation" is not useful. Rather, the teacher or calculator are used to check and ratify answers. That Rachel does not see peers as a key component in her mathematics learning is further exemplified in the following interview excerpt:

(Rachel)
I: So do you think it's important to try to help your neighbour or what?

R: I only try to help, I only help my neighbour when I'm told to by the teacher and we're allowed to because sometimes we're not allowed to and we could get our name on the board. Like say if it was a test. Then we aren't allowed to help our neighbour.

I: What about today? Do you think it would have been okay to help your neighbour?

R: I'm not really sure. So. And I didn't take the risk, so I didn't. Cause I wasn't sure.

I: Okay. But there are times when you're told to help your neighbour, is that right?

R: Yes.

I: Do you think that is helpful?

R: Yes, because it's a partner activity most of the time. And also I feel that Eddie needs quite a bit of help, cause you know, he's one of those boys which is quite naughty, and I feel that, I feel that I should try to help

him to get better. And I'm helping him with stuff. Like stuff that he doesn't understand and that the teacher won't answer for him. And I only answer it if it's for a specific reason. For instance if it was a test then I wouldn't answer it. I would just tell him one way that he could try to find out the answer. . . . Well most of the time my neighbour, which is Eddie, most of the time I don't believe him because he's a bit silly. So sometimes he says hey that's not, that's not right. Look at mine. Mine is right and yours is wrong. So sometimes I believe him and sometimes I don't. And most of the time when I do believe him he, I'm usually wrong. . . .

I: Are there other people in your class that might say things that might make you change your mind?

R: Well it depends who the person is. Like say it was somebody that wasn't very good at mathematics then I probably wouldn't listen to them. But say if it was someone good at their mathematics then I probably would listen to them.

Rachel clearly states that she does not see it of any value to her to take her neighbour's (Eddie's) views into consideration. It is interesting to note that her perspective on the notion of peer consultation is highly intertwined with the larger context of the social norms of the classroom. That is, Rachel's views of both her own and other students' roles in sharing ideas and helping one another are influenced by the behaviour and action rules of the classroom. She will not "take the risk" of helping another student unless she has been informed by the teacher that it is an acceptable or appropriate action for that mathematics lesson or point in the lesson. The same awareness of and adherence to the classroom rules was evident in the interview with Walter, although he does not express the same devaluing of peer assistance:

(Walter)
I: What were you thinking there?

W: I was thinking that I was going to ask Mr Y whether Allan won. But I shouldn't call out.

I: Why not?

W: Um because the rule is to put your hand up instead of calling out. Because if everyone called out you'd be, there'd be a loud noise. . . . Um because if lots of people knew the answer and everyone called out the answer cause they wanted Mr Y to know that they knew it, there'd be a big loud noise again and then all the other classes couldn't work properly.

In relation to classroom sharing of ideas, Anna explicitly expressed a dislike, rather than a devaluing, of the role of peer interaction. This dislike can be seen in the following interchange with the interviewer:

(Anna)
A: Our class is very noisy.

I: What do you think about that?

A: That the class could be more quieter.

I: Would that be good or bad, to be quiet?

A: Good.

I: Why?

A: I get headaches in class. . . . Cause it's too noisy.

I: Do you think being noisy has an effect upon what people are learning?

A: Yup. Because when they're talking they don't concentrate. And um they're not learning anything. And um it disturbs others. And I don't know why else.

I: Are there other things about class you'd like to comment on?

A: . . . Um. I reckon our class should be a bit more quieter because um no one learns if everyone is talking.

This excerpt reveals that Anna believes that people cannot learn when they are talking. In other words, from Anna's point of view, learning occurs when people are quiet and are concentrating. Thus, although she does not explicitly state it, Anna does not see value for her mathematics learning in peer interaction, or sharing or discussing ideas.

Although the discussion thus far has highlighted the absence of students valuing the role of peers in their mathematics learning, there was also evidence that students on occasion did help one another. For example:

(Amanda)
And um Sally and I were figuring out some of the time. Well if sort of like she got stuck on something I would show an example and help her. Like how I did that.

(Yvonne)
I: Do you think the other students in your class learned the same thing?

Y: Some already knew it, like the bright kids like know, but they just like, they sort of like lead the not so bright kids along.

I: In what way?

Y: Um like they always like put their hand up and answer the teacher and the other kids learn.

Amanda speaks of helping out one of her neighbours, while Yvonne mentions paying attention to responses of students in the class as a means by which her learning is sometimes guided. The students who she is helped by, by paying particular attention to them, are those she perceives to be "the bright kids". Interestingly, Rachel also referred to listening to and being helped by a peer if it was someone she perceived to be "good at their mathematics".

In summary, it can be noted that the specific role of peers within the classroom and the role of each student in relation to his or her peers, was clearly subordinate to that of the teacher. That is, as with the individual level, the teacher was construed by students to be the primary authority to determine legitimacy or correctness of procedures or answers.

CONCLUSIONS

Within the social realm of the two year five classes studied the process of "negotiation" of meaning was such that the conjecturing, criticising, explaining, testing and refining of ideas and procedures was primarily the responsibility of the teacher. The role of students within this "negotiation" process was minimal in comparison to that of the teacher, so that students did not regard themselves or peers as a source of mathematical knowledge. Although some group consensus occurred via group majority agreement on procedures or answers, a metaphor of classroom meaning making as "negotiation" could be said to be inappropriate for these two year five classes. A more appropriate descriptor is "ratification" in that, although ratification incorporates a notion of acceptance or agreement, it primarily denotes a rite of endorsement and approval to officially validate the agreement.

That these notions of endorsement and agreement are more appropriate descriptors than negotiation indicates a need to examine the intents and expectations of both the teachers and the students within the classroom processes of these two classes What are a teacher's intents in teaching a particular lesson, or in teaching mathematics in general? What are her beliefs and experiences in relation to mathematics learning and what are her expectations of students? What do the students' expect the teacher to do or say, and what do they believe are his intents and expectations for learning? These questions and related ones must be addressed before classroom processes can be interpreted in ways which capture the classroom contexts as experienced by participants themselves. For example, a number of students in this study expected the teacher to provide clear explanations of how things work and what to do with them. They expected the teacher to prescribe procedures, and hence, the issue of negotiation was not an issue at all. That is, the students expected the teaching to be what educators call direct instruction. Direct instruction has a number of

practical and effective educational features, but the issue of negotiation is not an anticipated component of such a mode of delivery.

This last point highlights that the concept of negotiation requires intention to negotiate, which in turn highlights the role of classroom social contexts. The social rules and norms of a classroom, and therefore the resulting classroom practices, are necessarily key components of ways in which students construct mathematical meaning. For example, if peer discussion is neither valued nor encouraged, then it is unlikely students will see it as either a necessary or important component of their learning. Similarly, if students are not provided opportunities to develop skills at verbalising their mathematics understandings, then it is inappropriate to expect them to have capacities to do so, even if one argues that developmental age level also might be a relevant factor.

Further, if the classroom context is such that procedural knowledge and rule following are emphasised, or are sufficient to attain what is considered "success in mathematics" in a particular classroom, then why would a student aim for conceptual understanding? In a context in which procedures and correct answers are the main goals, rule following is undoubtedly a viable goal and practice for students. Students therefore could be said to be constructing their mathematical meanings in ways which are viable within the social contexts of their classrooms experiences, with viability arising from "the action of an individual and the extent to which those actions facilitate the attainment of goals in the social contexts of actions" (Tobin & Tippins, 1993; p.5).

This notion of constructing viable knowledge and actions is an aspect of constructivist theories. However, since teachers generally want students to learn with "understanding", if viability from a student's perspective is determined by ratification or endorsement, then the nature of what students learn is problematic. It is problematic in that what teachers and students see as mathematical understanding and the intended outcomes of mathematics lessons are possibly in discord with each other or with actual classroom practices. Vital to these issues are teachers' and students' beliefs about mathematics learning and the nature of mathematics, for these beliefs will shape how they see "understanding". Thus, if construction of mathematical meaning is to be an object of research, then an integral component of examination must be individuals' beliefs about the nature of mathematics and what constitutes mathematics learning. Only then will educators be equipped to appropriately and adequately study mathematics classrooms in ways which might enlighten or transform mathematics education.

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